Comments on "Tight Bounds and Invertible Average Error Probability Expressions over Composite Fading Channels"

Seong Ho Chae and Hoojin Lee

Abstract: **In the paper "Tight Bounds and Invertible Average Error Probability Expressions over Composite Fading Channels" of Wang *et al.* [1], they derived the algebraic formulas of upper- and lower- bounds for the average symbol error probability (ASEP) of M-ary phase shift keying (M-PSK) in a class of composite fading channels by using mixture gamma (MG) distribution, which is based on the assumption of the lack of closed-form expression for exact ASEP. However, the exact ASEP can be represented in closed-form by utilizing hypergeometric functions. Therefore, in this paper, we formulate the closed-form formulas of the exact and asymptotic ASEPs for general M-PSK in a class of the composite fading channels over MG distribution. We present some numerical results to validate our derived analytical results.**

Index Terms: **M-ary phase shift keying, mixture gamma distribution, symbol error probability**

I. INTRODUCTION

RECENTLY , there have been growing interests in evaluating the wireless communication performance by modeling the composite fading channels (multipath fading and shadowing) as mixture gamma (MG) distribution. The MG distribution can provide the tractable form for many composite fading channels, which facilitates to analyze the error probability or channel capacity [1], [2]. Thus, Wang *et al.* [1] analyzed the average symbol error probability (ASEP) of M-ary phase shift keying (M-PSK) by approximating the signal-to-noise (SNR) distribution of composite fading channels with MG distribution. However, due to the lack of the closed-form expression for exact ASEP (i.e., (9) in [1]), they attempted to derive the tight algebraic expressions of upper bounds for ASEP of M-PSK when $M > 2$ and lower- and upper- bounds when $M = 2$, particularly, under Nakagami-m fading, Generalized-K (K_G) fading, and Nakagami-lognormal (NL) fading. However, we find that the closed-form expression for the exact ASEP is available by applying Appell and Gauss hypergeometric functions. Therefore, motivated by this aspect, the aim of this paper is to appropriately derive the closed-form formulas for the exact ASEP along with the corresponding asymptotic expressions in high SNR regime for general M-PSK in a class of the composite fading channels.

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S. H. Chae is with the Department of Electronics Engineering, Korea Polytechnic University, Siheung, 15073, South Korea, email: shchae@kpu.ac.kr.

H. Lee is with Division of IT Convergence Engineering, Hansung University, Seoul, 02876, South Korea, email: hjlee@hansung.ac.kr.

H. Lee is the corresponding author.

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II. EXISTING EXACT BUT INTEGRAL FORM EXPRESSIONS OF ASEP FOR M-PSK [1]

The MG distribution to approximate the probability density function of SNR γ is given by [1, (3)]

$$f_\gamma(x) = \sum_{i=1}^{N} \alpha_i x^{\beta_i - 1} e^{-\zeta_i x},\ x \geq 0, \tag{1}$$

and the exact ASEP for any M-PSK over the MG distribution is given in integral form (i.e., not in closed-form) by [1, (9)]

$$P(\bar{\gamma}) = \sum_{i=1}^{N} \frac{\alpha_i \Gamma(\beta_i)}{\pi \zeta_i^{\beta_i}} \int_0^{\frac{(M-1)\pi}{M}} \left(\frac{\sin^2\theta}{\sin^2\theta + \frac{\sin^2\left(\frac{\pi}{M}\right)}{\zeta_i}} \right)^{\beta_i} d\theta, \tag{2}$$

where $\alpha_i, \beta_i, \zeta_i$ denote the parameters of i-th MG component depending on the different fading models, N is the number of terms, and $\Gamma(\cdot)$ denotes the gamma function represented by $\Gamma(t) = \int_0^\infty x^{t-1} e^{-x} dx$.

By employing the MG distribution, the exact ASEP expressions for M-PSK over various fading channels can be represented in integral form [1]. For examples, the exact ASEP expressions under Nakagami-m, K_G, and NL composite fading channels are expressed as follows.

A. Nakagami-m Fading

The parameters for Nakagami-m fading are given by

$$N = 1,\ \alpha_1 = \frac{m^m}{\Gamma(m)\bar{\gamma}^m},\ \beta_1 = m,\ \zeta_1 = \frac{m}{\bar{\gamma}}, \tag{3}$$

and the exact ASEP expression for any M-PSK is given by [1, (26)]

$$P^{\text{Nakagami}-m}(\bar{\gamma}) = \frac{1}{\pi} \int_0^{\frac{(M-1)\pi}{M}} \left(\frac{\sin^2\theta}{\sin^2\theta + \frac{\sin^2\left(\frac{\pi}{M}\right)}{m}\bar{\gamma}} \right)^{m} d\theta. \tag{4}$$

B. K_G Fading

The parameters for K_G fading are given by [1, (27)]

$$\alpha_i = \psi(\theta_i, \beta_i, \zeta_i),\ \beta_i = m,\ \zeta_i = \frac{\lambda}{t_i},$$
$$\theta_i = \frac{\lambda^m \omega_i t_i^{l-m-1}}{\Gamma(m)\Gamma(l)},\ \lambda = \frac{lm}{\bar{\gamma}}, \tag{5}$$

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and the exact ASEP expression for any M-PSK is given by

$$P^{K_G}(\bar{\gamma}) = \sum_{i=1}^{N} \frac{w_i t_i^{l-1}}{\sum_{j=1}^{N} w_j t_j^{l-1}} \frac{1}{\pi} \int_0^{\frac{(M-1)\pi}{M}} \left(\frac{\sin^2\theta}{\sin^2\theta + \frac{\sin^2\left(\frac{\pi}{M}\right) t_i}{lm}\bar{\gamma}} \right)^m d\theta, \quad (6)$$

where $\psi(\theta_i, \beta_i, \zeta_i) = \theta_i / (\sum_{j=1}^{N} \theta_j \Gamma(\beta_j) \zeta_j^{-\beta_j})$, t_i and ω_i are the abscissas and weight factors for Gaussian-Laguerre integration [3], and l and m are shaping parameters representing the multipath fading and shadowing effects of the wireless channel.

C. NL Composite Fading

The parameters for NL composite fading are given by

$$\alpha_i = \psi(\theta_i, \beta_i, \zeta_i),\ \beta_i = m,\ \zeta_i = \frac{m}{\rho} e^{-(\sqrt{2}\sigma t_i + \mu)},$$
$$\theta_i = \left(\frac{m}{\rho}\right)^m \frac{\omega_i e^{-m(\sqrt{2}\sigma t_i + \mu)}}{\sqrt{\pi}\Gamma(m)},\ \rho = C\bar{\gamma} \quad (7)$$

and the exact ASEP expression for any M-PSK is given by [1, (28)]

$$P^{\mathrm{NL}}(\bar{\gamma}) = \sum_{i=1}^{N} \frac{w_i}{\sum_{j=1}^{N} w_j} \frac{1}{\pi} \int_0^{\frac{(M-1)\pi}{M}} \left(\frac{\sin^2\theta}{\sin^2\theta + \frac{\sin^2\left(\frac{\pi}{M}\right) C}{m e^{-(\sqrt{2}\sigma t_i + \mu)}}\bar{\gamma}} \right)^m d\theta, \quad (8)$$

where $C = (\sum_{j=1}^{N} w_j) / (\sum_{i=1}^{N} w_i e^{\sqrt{2}\sigma t_i + \mu})$, m is the fading parameter in Nakagami-m fading, μ and σ are the mean and standard deviation of the lognormal distribution, respectively, and t_i and ω_i are the abscissas and weight factors for Gaussian-Hermite integration [3].

III. CLOSED-FORM EXPRESSIONS OF EXACT AND ASYMPTOTIC ASEP FOR M-PSK

In this section, we derive the exact and asymptotic ASEPs in closed-form for general M-PSK.

A. Exact Closed-form ASEP Expressions for M-PSK

The ASEP in (2) can be reformulated as

$$P(\bar{\gamma}) = \sum_{i=1}^{N} \frac{\alpha_i \Gamma(\beta_i)}{\zeta_i^{\beta_i}} \cdot I_{\beta_i}\left(\frac{\sin^2\left(\frac{\pi}{M}\right)}{\zeta_i}, \frac{(M-1)\pi}{M} \right), \quad (9)$$

where

$$I_n(z, \varphi) = \frac{1}{\pi} \int_0^{\varphi} \left(\frac{\sin^2\theta}{\sin^2\theta + z} \right)^n d\theta. \quad (10)$$

Since M is the modulation order of M-PSK and must be a positive integer power of 2, e.g., $M = 2, 4, 8, 16, \cdots$, the relationship of $\frac{\pi}{2} \le \varphi \left(= \frac{(M-1)\pi}{M}\right) \le \pi$ always holds. Therefore, for $\pi/2 \le \varphi \le \pi$, (10) can be written in closed-form by using (5) in [4] as

$$I_n(z, \varphi) = \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \left(\frac{\sin^2\theta}{\sin^2\theta + z} \right)^n d\theta + \frac{1}{\pi} \int_{\frac{\pi}{2}}^{\varphi} \left(\frac{\sin^2\theta}{\sin^2\theta + z} \right)^n d\theta. \quad (11)$$

The first term in (11) can be re-written by changing the variable $t = \sin^2\theta$ and after some arithmetic manipulations with the relationship [6, (3.211)] of

$$\int_0^1 x^{\lambda-1}(1-x)^{\mu-1}(1-\mu x)^{-\varrho}(1-vx)^{-\sigma} dx = B(\mu, \lambda) F_1(\lambda; \varrho, \sigma; \lambda + \mu; u, v), \quad (12)$$

as

$$\frac{1}{\pi} \int_0^{\frac{\pi}{2}} \left(\frac{\sin^2\theta}{\sin^2\theta + z} \right)^n d\theta = \frac{1}{2\pi z^n} \int_0^1 t^{n-\frac{1}{2}} (1-t)^{-1/2} \left(1 + \frac{t}{z}\right)^{-n} dt \quad (13)$$
$$= \frac{B(1, n + (1/2))}{2\pi z^n} F_1\left(n + \frac{1}{2}; \frac{1}{2}, n; n + \frac{3}{2}; 1, -\frac{1}{z}\right), \quad (14)$$

where $F_1(a; b_1, b_2; c; x_1, x_2)$ denotes the Appell hypergeometric function [5, (07.36.02.0001.01)] and $B(a, b)$ denotes the beta function. Similarly, the second term in (11) can be re-written by changing the variable $t = \cos^2\theta / \cos^2\varphi$ and after some arithmetic manipulations as

$$\frac{1}{\pi} \int_{\frac{\pi}{2}}^{\varphi} \left(\frac{\sin^2\theta}{\sin^2\theta + z} \right)^n d\theta = \frac{\cos\varphi}{\pi (1+z)^n} F_1\left(\frac{1}{2}; \frac{1}{2} - n, n; \frac{3}{2}; \cos^2\varphi, \frac{\cos^2\varphi}{1+z}\right). \quad (15)$$

Therefore, (11) can be written by

$$I_n(z, \varphi) = \frac{B(1, n + (1/2))}{2\pi z^n} F_1\left(n + \frac{1}{2}; \frac{1}{2}, n; n + \frac{3}{2}; 1, -\frac{1}{z}\right) - \frac{\cos\varphi}{\pi (1+z)^n} F_1\left(\frac{1}{2}; \frac{1}{2} - n, n; \frac{3}{2}; \cos^2\varphi, \frac{\cos^2\varphi}{1+z}\right). \quad (16)$$

Consequently, the exact closed-form ASEP for M-PSK in (9) can be expressed as (17).

In the following, we provide the closed-form expressions of ASEP for Nakagami-m, K_G, and NL composite fading channels.

A.1 Nakagami-m Fading

For Nakagami-m fading, the exact ASEP of M-PSK can be expressed in closed-form as (18).

A.2 K_G Fading

For K_G fading, the exact ASEP of M-PSK can be represented in closed-form as (19).

A.3 NL Composite Fading

For NL composite fading, the exact ASEP of M-PSK can be given in closed-form as (20).

B. Asymptotic Closed-form ASEP Expressions for M-PSK

From (3), (5), and (7), we see that ζ_i can be re-represented in the form of $\zeta_i = (c_i \bar{\gamma})^{-1}$, where c_i is a constant depending

$$P(\bar{\gamma}) = \sum_{i=1}^{N} \frac{\alpha_i \Gamma(\beta_i)}{\zeta_i^{\beta_i}} \left[\frac{B(1,\beta_i+\frac{1}{2})}{2\pi \left(\frac{\sin^2\left(\frac{\pi}{M}\right)}{\zeta_i}\right)^{\beta_i}} F_1 \left(\beta_i + \frac{1}{2}; \frac{1}{2}, \beta_i; \beta_i + \frac{3}{2}; 1, -\frac{\zeta_i}{\sin^2\left(\frac{\pi}{M}\right)}\right) \right.$$
$$\left. - \frac{\cos\left(\frac{(M-1)\pi}{M}\right)}{\pi\left(1+\frac{\sin^2\left(\frac{\pi}{M}\right)}{\zeta_i}\right)^{\beta_i}} F_1 \left(\frac{1}{2}; \frac{1}{2}-\beta_i, \beta_i; \frac{3}{2}; \cos^2\left(\frac{(M-1)\pi}{M}\right), \frac{\cos^2\left(\frac{(M-1)\pi}{M}\right)}{1+\frac{\sin^2\left(\frac{\pi}{M}\right)}{\zeta_i}}\right)\right] \quad (17)$$

$$P^{\text{Nakagami}-m}(\bar{\gamma}) = \frac{B(1,m+\frac{1}{2})}{2\pi\left(\frac{\bar{\gamma}}{m}\sin^2\left(\frac{\pi}{M}\right)\right)^m} F_1\left(m+\frac{1}{2}; \frac{1}{2}, m; m+\frac{3}{2}; 1, -\frac{m}{\bar{\gamma}\sin^2\left(\frac{\pi}{M}\right)}\right)$$
$$- \frac{\cos\left(\frac{(M-1)\pi}{M}\right)}{\pi\left(1+\frac{\bar{\gamma}}{m}\sin^2\left(\frac{\pi}{M}\right)\right)^m} F_1\left(\frac{1}{2}; \frac{1}{2}-m, m; \frac{3}{2}; \cos^2\left(\frac{(M-1)\pi}{M}\right), \frac{\cos^2\left(\frac{(M-1)\pi}{M}\right)}{1+\frac{\bar{\gamma}}{m}\sin^2\left(\frac{\pi}{M}\right)}\right) \quad (18)$$

$$P^{K_G}(\bar{\gamma}) = \sum_{i=1}^{N} \frac{w_i t_i^{l-1}}{\sum_{j=1}^{N} w_j t_j^{l-1}} \left[\frac{B(1,m+\frac{1}{2})}{2\pi\left(\frac{\sin^2\left(\frac{\pi}{M}\right)t_i\bar{\gamma}}{lm}\right)^m} F_1\left(m+\frac{1}{2}; \frac{1}{2}, m; m+\frac{3}{2}; 1, -\frac{lm}{\sin^2\left(\frac{\pi}{M}\right)t_i\bar{\gamma}}\right)\right.$$
$$\left. - \frac{\cos\left(\frac{(M-1)\pi}{M}\right)}{\pi\left(1+\frac{\sin^2\left(\frac{\pi}{M}\right)t_i\bar{\gamma}}{lm}\right)^m} F_1\left(\frac{1}{2}; \frac{1}{2}-m, m; \frac{3}{2}; \cos^2\left(\frac{(M-1)\pi}{M}\right), \frac{\cos^2\left(\frac{(M-1)\pi}{M}\right)}{1+\frac{\sin^2\left(\frac{\pi}{M}\right)t_i\bar{\gamma}}{lm}}\right)\right] \quad (19)$$

$$P^{\text{NL}}(\bar{\gamma}) = \sum_{i=1}^{N} \frac{w_i}{\sum_{j=1}^{N} w_j} \left[\frac{B(1,m+\frac{1}{2})}{2\pi\left(\frac{\sin^2\left(\frac{\pi}{M}\right)\rho}{me^{-(\sqrt{2}\sigma t_i+\mu)}}\right)^m} F_1\left(m+\frac{1}{2}; \frac{1}{2}, m; m+\frac{3}{2}; 1, -\frac{me^{-(\sqrt{2}\sigma t_i+\mu)}}{\sin^2\left(\frac{\pi}{M}\right)\rho}\right)\right.$$
$$\left. - \frac{\cos\left(\frac{(M-1)\pi}{M}\right)}{\pi\left(1+\frac{\sin^2\left(\frac{\pi}{M}\right)\rho}{me^{-(\sqrt{2}\sigma t_i+\mu)}}\right)^m} F_1\left(\frac{1}{2}; \frac{1}{2}-m, m; \frac{3}{2}; \cos^2\left(\frac{(M-1)\pi}{M}\right), \frac{\cos^2\left(\frac{(M-1)\pi}{M}\right)}{1+\frac{\sin^2\left(\frac{\pi}{M}\right)\rho}{me^{-(\sqrt{2}\sigma t_i+\mu)}}}\right)\right] \quad (20)$$

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on the channel fading. Accordingly, the ASEP in (9) can be re-written by

$$P(\bar{\gamma}) = \sum_{i=1}^{N} \alpha_i \Gamma(\beta_i)(c_i\bar{\gamma})^{\beta_i} I_{\beta_i}\left(K_i\bar{\gamma}, \phi\right), \quad (21)$$

where $K_i = c_i \sin^2\left(\frac{\pi}{M}\right)$ and $\phi = \frac{(M-1)\pi}{M}$. If $\bar{\gamma} \to \infty$, then $\zeta_i \to 0$ and $F_1\left(a; b_1, b_2; c; x_1, 0\right)$ is simplified to ${}_2F_1\left(a, b_1; c; x_1\right)$, where ${}_2F_1\left(a, b; c; x\right)$ is the Gauss hypergeometric function [5, (07.23.02.0001.01)]. Thus, through the high SNR approximation (i.e., $\bar{\gamma} \to \infty$), $I_{\beta_i}\left(K_i\bar{\gamma}, \phi\right)$ in (21) can be simplified as

$$\lim_{\bar{\gamma}\to\infty} I_n(K_i\bar{\gamma}, \phi) = \xi_n(K_i, \phi) \cdot \bar{\gamma}^{-n}, \quad (22)$$

where

$$\xi_n(K_i, \varphi) = \frac{\Gamma(n+\frac{1}{2})}{2\sqrt{\pi}\Gamma(n+1)(K_i)^n} - \frac{\cos\phi}{\pi(K_i)^n}\, {}_2F_1\left(\frac{1}{2}, \frac{1}{2}-n; \frac{3}{2}; \cos^2\phi\right). \quad (23)$$

Therefore, the following compact asymptotic ASEP formula can be obtained as

$$P_{\bar{\gamma}\to\infty} = \sum_{i=1}^{N} \alpha_i \Gamma(\beta_i)(c_i)^{\beta_i} \left[\frac{\Gamma(n+\frac{1}{2})}{2\sqrt{\pi}\Gamma(n+1)(K_i)^n} - \frac{\cos\phi}{\pi(K_i)^n}\, {}_2F_1\left(\frac{1}{2}, \frac{1}{2}-n; \frac{3}{2}; \cos^2\phi\right)\right]. \quad (24)$$

B.1 Nakagami-m Fading

For Nakagami-m fading, $c_i = 1/m$, $K_i = \frac{1}{m}\sin^2\left(\frac{\pi}{M}\right)$, and the asymptotic ASEP for high SNR can be expressed as

$$P_{\bar{\gamma}\to\infty}^{\text{Nakagami}-m} = \left[\frac{\Gamma(m+\frac{1}{2})}{2\sqrt{\pi}\Gamma(m+1)\left(\frac{1}{m}\sin^2\left(\frac{\pi}{M}\right)\right)^m} - \frac{\cos\left(\frac{(M-1)\pi}{M}\right)\, {}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\cos^2\left(\frac{(M-1)\pi}{M}\right)\right)}{\pi\left(\frac{1}{m}\sin^2\left(\frac{\pi}{M}\right)\right)^m}\right]\bar{\gamma}^{-m}. \tag{25}$$

B.2 K_G Fading

For K_G fading, $c_i = \frac{t_i}{lm}$, $K_i = \frac{t_i}{lm}\sin^2\left(\frac{\pi}{M}\right)$, and the asymptotic ASEP for high SNR can be presented as

$$P_{\bar{\gamma}\to\infty}^{K_G} = \sum_{i=1}^{N}\frac{w_i t_i^{l-1}}{\sum_{j=1}^{N} w_j t_j^{l-1}}\left[\frac{\Gamma(m+\frac{1}{2})}{2\sqrt{\pi}\Gamma(m+1)\left(\frac{t_i}{lm}\sin^2\left(\frac{\pi}{M}\right)\right)^m} - \frac{\cos\left(\frac{(M-1)\pi}{M}\right)\, {}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\cos^2\left(\frac{(M-1)\pi}{M}\right)\right)}{\pi\left(\frac{t_i}{lm}\sin^2\left(\frac{\pi}{M}\right)\right)^m}\right]\bar{\gamma}^{-m}. \tag{26}$$

B.3 NL Composite Fading

For NL composite fading, $c_i = \frac{Ce^{\sqrt{2}\sigma t_i+\mu}}{m}$, $K_i = \frac{Ce^{\sqrt{2}\sigma t_i+\mu}}{m}\sin^2\left(\frac{\pi}{M}\right)$, $C = \frac{\sum_{j=1}^{N} w_j}{\sum_{i=1}^{N} w_i e^{\sqrt{2}\sigma t_i+\mu}}$, and the asymptotic ASEP in high SNR can be formulated as

$$P_{\bar{\gamma}\to\infty}^{\text{NL}} = \sum_{i=1}^{N}\frac{w_i}{\sum_{j=1}^{N} w_j}\left[\frac{\Gamma(m+\frac{1}{2})}{2\sqrt{\pi}\Gamma(m+1)\left(\frac{Ce^{\sqrt{2}\sigma t_i+\mu}}{m}\sin^2\left(\frac{\pi}{M}\right)\right)^m} - \frac{\cos\left(\frac{(M-1)\pi}{M}\right)\, {}_2F_1\left(\frac{1}{2},\frac{1}{2}-m;\frac{3}{2};\cos^2\left(\frac{(M-1)\pi}{M}\right)\right)}{\pi\left(\frac{Ce^{\sqrt{2}\sigma t_i+\mu}}{m}\sin^2\left(\frac{\pi}{M}\right)\right)^m}\right]\bar{\gamma}^{-m}. \tag{27}$$

IV. NUMERICAL RESULTS

In this section, we evaluate the average symbol error probability of M-PSK to validate our analytical results in the previous section by using MATLAB software. Specifically, we compare the Monte-Carlo ASEP, the exact but integral form ASEP, the proposed exact closed-form ASEP, and the asymptotic ASEP in high SNR of M-PSK for three fading channels (e.g., Nakagami-m fading, K_G fading, and NL composite fading) and examine how modulation order M and channel parameters affect on the ASEPs. We consider the system parameters as $m = 2$, $(m, l, N) = (2, 5, 10)$, and $(m, \sigma, \mu, N) = (2, 1, 0.25, 9)$ for Nakagami-m fading, K_G composite fading, and NL composite fading, respectively [1].

Fig. 1 plots the Monte-Carlo ASEP, the exact ASEP (4), closed-form ASEP (18), and asymptotic ASEP (25) of M-PSK under Nakagami-m fading versus average SNR $\bar{\gamma}$ (dB) for various M, where $m = 2$. Fig. 2 depicts the Monte-Carlo ASEP, the exact ASEP (6), closed-form ASEP (19), and asymptotic ASEP (26) of M-PSK under K_G fading versus average SNR $\bar{\gamma}$ (dB) for various M, where $(m, l, N) = (2, 5, 10)$. Fig. 3 illustrates the Monte-Carlo ASEP, the exact ASEP (8), closed-form ASEP (20), and asymptotic ASEP (27) of M-PSK under NL composite fading versus average SNR $\bar{\gamma}$ (dB) for various M, where $(m, \sigma, \mu, N) = (2, 1, 0.25, 9)$. We can clearly find that all the Figures 1-3 validate that the derived exact closed-form ASEPs for three fading channels completely match well to the exact but integral form ASEPs in all average SNR regions and the derived asymptotic ASEPs also match well to the exact ASEPs in high SNR region. Those figures commonly showed that the ASEP increases as the modulation order M increases and it decreases as average SNR increases.

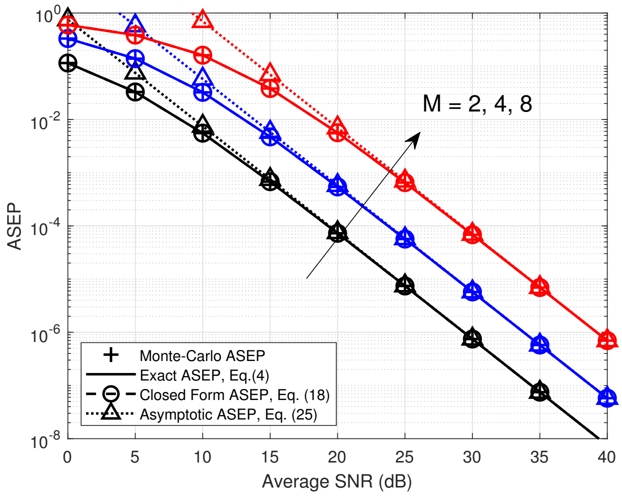

Fig. 1. Comparison of ASEPs (Monte-Carlo ASEP, exact ASEP (4), closed-form ASEP (18), and asymptotic ASEP in high SNR (25)) of M-PSK under Nakagami-m fading versus average SNR $\bar{\gamma}$ (dB) for various M, where $m = 2$.

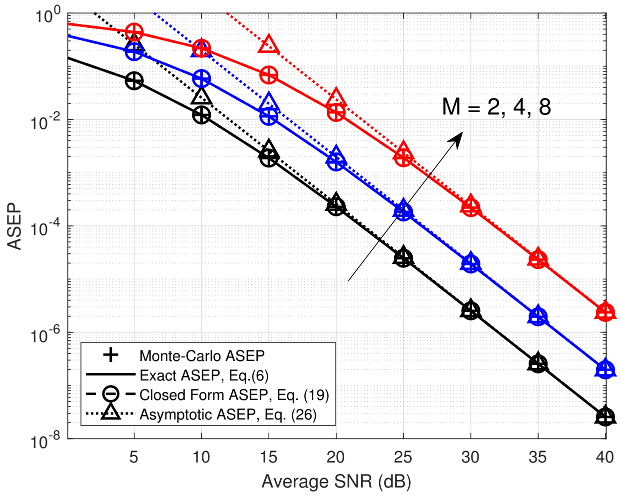

Fig. 2. Comparison of ASEPs (Monte-Carlo ASEP, exact ASEP (6), closed-form ASEP (19), and asymptotic ASEP in high SNR (26)) of M-PSK under K_G fading versus average SNR $\bar{\gamma}$ (dB) for various M, where $(m, l, N) = (2, 5, 10)$.

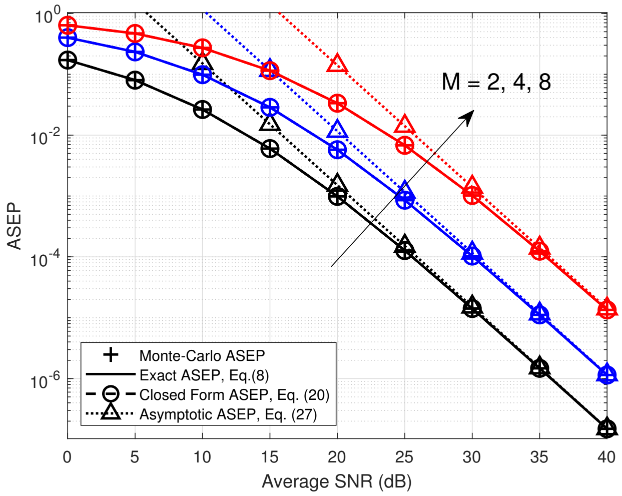

Fig. 3. Comparison of ASEPs (Monte-Carlo ASEP, exact ASEP (8), closed-form ASEP (20), and asymptotic ASEP in high SNR (27)) of M-PSK under NL composite fading versus average SNR $\bar{\gamma}$ (dB) for various M, where $(m, \sigma, \mu, N) = (2, 1, 0.25, 9)$.

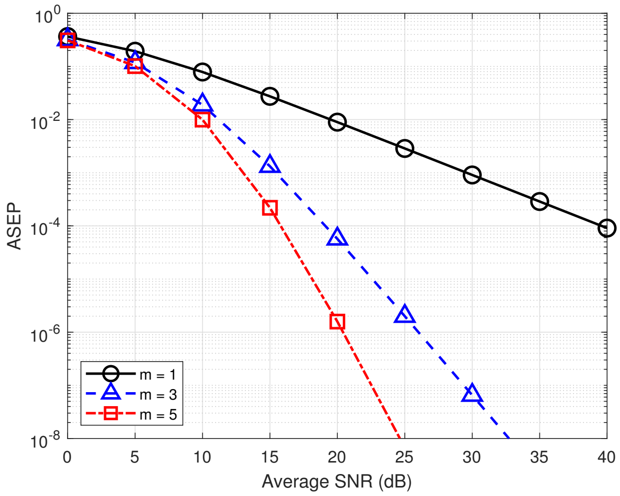

Fig. 4. Comparison of ASEP of QPSK under Nakagami-m fading versus average SNR $\bar{\gamma}$ (dB) for various m.

Fig. 4 compares the ASEP of QPSK under Nakagami-m fading versus average SNR $\bar{\gamma}$ (dB) for various m, which shows that the ASEP decreases as m increases. This is because the fading severity of the channel decreases as m increases. Fig. 5 compares the ASEP of QPSK under generalized-$K(K_G)$ fading versus average SNR $\bar{\gamma}$ (dB) for various m and l, where l and m are shaping parameters representing the multipath fading and shadowing effects of the wireless channel. This figure shows that the ASEP decreases as l and m increase. This is because the multipath fading and shadowing effects of the wireless channel decrease as l and m increase. Fig. 6 compares the ASEP of QPSK under Nakagami-lognormal(NL) fading versus average SNR $\bar{\gamma}$ (dB) for various σ, where σ is the standard deviation of the lognormal distribution. This figure shows that the ASEP increases as σ increases. This is because the lognormal shadowing effect increases as σ increases.

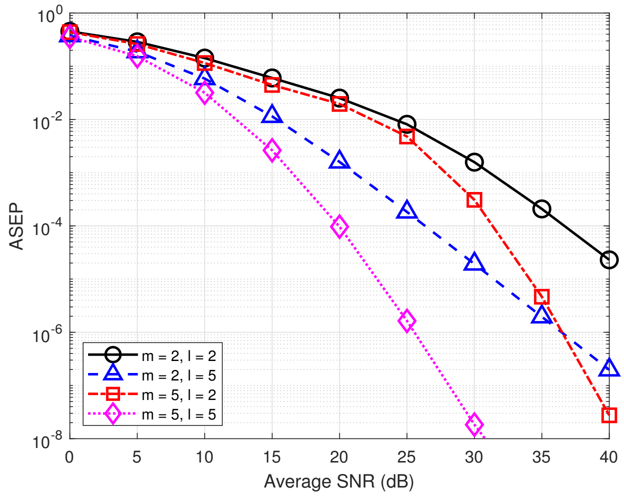

Fig. 5. Comparison of ASEP of QPSK under K_G fading versus average SNR $\bar{\gamma}$ (dB) for various m and l, where $N = 10$.

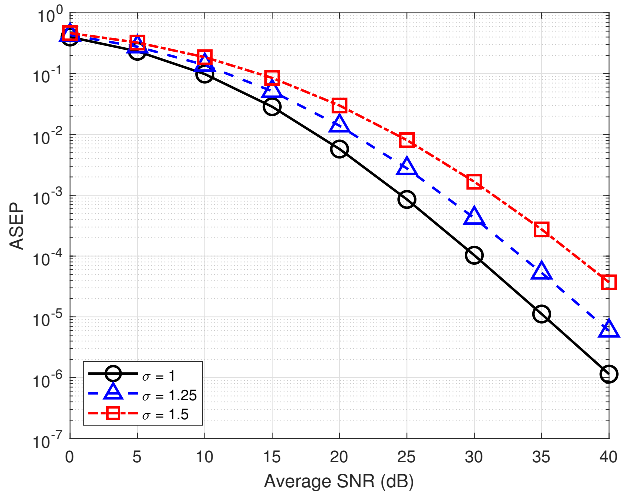

Fig. 6. Comparison of ASEP of QPSK under NL composite fading versus average SNR $\bar{\gamma}$ (dB) for various σ, where $(m, \mu, N) = (2, 0.25, 9)$.

V. CONCLUSION

In this paper, we have newly derived the closed-form formulas for the exact and asymptotic ASEPs of general M-PSK over various composite fading channels approximated by mixture gamma distribution. Specifically, we have represented the exact and asymptotic closed-form ASEP expressions by judiciously adopting Appell and Gauss hypergeometric functions, respectively, which enable us to obtain more explicit insights into the achievable ASEP performance over a variety of fading channels.

REFERENCES

[1] Q. Wang, H. Lin, and P.-Y. Kam, "Tight bounds and invertible average error probability expressions over composite fading channels," *IEEE/KICS J. Commun. Netw.*, vol. 18, no. 2, pp. 182-189, Apr. 2016.

[2] S. Atapattu, C. Tellambura, and H. Jiang, "A mixture gamma distribution to model the SNR of wireless channels," *IEEE Trans. Wireless Commun.*, vol. 10, no. 12, pp. 4193-4203, Dec. 2011.

[3] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions: With Formulus, Graphs, and Mathematical Tables*. Courier Corporation, 1964.

[4] H. Lee, "Exact and asymptotic BER analysis of 2x2 FRLR-STBC with BPSK modulation," *Electronics Letters*, vol. 53, no. 11, pp. 720-722, May 2017.

[5] The Wolfram Functions Site. Accessed: Feb. 2020. [Online]. Available: http://functions.wolfram.com

[6] I. S. Gradshteyn and I. M. Ryzhik, "Table of integrals, series, and products", Academic Press, 7th edition, 2007.

Seong Ho Chae received the B.S. degree from the School of Electronic and Electrical Engineering, Sungkyunkwan University, Suwon, South Korea, in 2010, and the M.S. and Ph.D. degrees from the School of Electrical Engineering, Korea Advanced Institute of Science and Technology (KAIST), Daejeon, South Korea, in 2012 and 2016, respectively. He was a Post-Doctoral Research Fellow with KAIST in 2016 and a Senior Researcher with Agency for Defense Development from 2016 to 2018, where he was involved in research and development for military communication system and frequency management software. He is currently an Assistant Professor with Korea Polytechnic University. His research interests include stochastic geometry, edge caching and computing, UAV communication, and LEO satellites constellation.

Hoojin Lee received his B.S. degree from the School of Electrical Engineering, Seoul National University, Seoul, South Korea, in 1997, and his M.S. and Ph.D. degrees in Electrical and Computer Engineering from the University of Texas at Austin, Austin, TX, USA, in 2002 and 2007, respectively. From 2008 to 2009, he worked as a systems and architecture engineer in the Algorithm and Standards Team, Cellular Products Group of Freescale Semiconductor, Inc., Austin, TX, USA. Since 2009, he has been working at the Division of IT Convergence Engineering in Hansung University, Seoul, Korea, where he is currently a Professor. His current research interests are in the areas of signal processing, communication theory, physical-layer security for communication systems, etc.