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강한 대기 난기류 채널 환경에서의 IM/DD 및 OOK 기반의 FSO 시스템에 대한 닫힌 형태의 비트 오류율 수식 유도

(Novel Exact and Asymptotic Closed-form BER Expressions for FSO System with IM/DD and OOK over Strong Atmospheric Turbulence Channels)

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요 약

본 논문에서는 강한 대기 난기류 채널 환경에서 IM/DD (Intensity Modulation with Direct Detection) 및 OOK (On-Off Keying) 방식이 적용된 FSO (Free-Space Optical) 시스템에 대한 평균 비트 오류율 (Bit-Error Rate: BER)의 새로운 닫힌 형태의 수식 및 점근적 수식을 함께 유도한다. 기존에 제시되어 있는 수식과 비교할 때, 본 논문에서 유도된 BER 수식들과 변조 이득 (Modulation Gain) 및 다이버시티 차수 (Diversity Order) 등은 FSO 시스템 성능에 대한 보다 직관적이고 유용한 통찰력을 제공한다. 수치 해석 결과를 통해 유도된 수식들의 정확성도 함께 검증한다.

Abstract

In this paper, we derive novel exact and asymptotic closed-form bit-error rate (BER) formulas for the free-space optical (FSO) system employing intensity modulation with direct detection (IM/DD) and on-off keying (OOK) modulation especially over strong atmospheric turbulence channels. The derived exact and asymptotic BER formulas as well as the corresponding asymptotic modulation gain and diversity order can give us more intuitive and useful insights into the achievable BER performance than the existing BER expression. Through some numerical results, the accuracy of our theoretical expressions is also corroborated.

Keywords: Free-space optical (FSO) communication, Bit-error rate (BER), Strong atmospheric turbulence

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Ⅰ. Introduction

Free-space optical (FSO) communications have gained enormous attention due to the remarkable advantages of high data rates, license-free spectrum, easy deployment, and high security. However, the performance of FSO systems can be significantly deteriorated by the atmospheric turbulence. Thus, various statistical models have been presented to effectively describe the FSO link statistics, which include Gamma-Gamma distribution, negative exponential distribution, *K*-distribution, and so on[1~6].

In this work, the FSO system utilizing intensity modulation with direct detection (IM/DD) and on-off keying (OOK) modulation is considered especially over strong atmospheric turbulence modeled by negative exponential distribution. In particular, this paper aims at deriving novel exact and asymptotic closed-form formulas of the average bit-error rate (BER). In fact, the exact BER expression is already given in [3]. But, it is not easy for us to gain any intuitive and useful insights into the achievable BER performance, since the existing BER expression given in [3] is formulated in terms of a complicated Meijer's *G*-function only. Therefore, in order to overcome the limitations of the existing formula, we derive novel and more intuitive formulas of the exact and asymptotic BER performances along with the asymptotic modulation gain and diversity order. Moreover, we also present some numerical results to substantiate the accuracy of the derived formulas.

Ⅱ. System and Channel Model

We consider the FSO communication system with IM/DD and OOK, where laser beams propagate along a horizontal path through a turbulence channel with additive white Gaussian noise (AWGN) and assuming that the channel is memoryless, stationary and ergodic, with independent and identically distributed (i.i.d.) intensity fading statistics. Then, the input-out relationship can be given by[1~3]

$$y = sx + n = \eta I x + n, \tag{1}$$

where y represents the received signal, η is the effective photo-current conversion ratio, I is the normalized irradiance, x is the modulated signal, and n denotes AWGN with zero mean and variance $\sigma_n^2 = N_0/2$.

Considering a limiting case of strong atmospheric turbulence conditions, the normalized irradiance intensity at the receiver can be assumed to follow the negative exponential distribution with the corresponding probability density function (pdf) as $f_I(I) = \exp(-I)$ with $I \geq 0$[3~6]. Then, the received instantaneous signal-to-noise ratio (SNR) can be defined as $\gamma = (\eta I)^2/N_0$, and the average SNR can be also expressed as $\bar{\gamma} = (\eta \bar{I})^2/N_0$, where $\bar{I}$ denotes the mean value of the irradiance. Thus, we can finally obtain the following pdf for γ as[3, 4]

$$f_\gamma(\gamma) = \frac{1}{2\sqrt{\gamma\bar{\gamma}}}\exp\left(-\sqrt{\frac{\gamma}{\bar{\gamma}}}\right). \tag{2}$$

Ⅲ. Existing Exact BER Formula

The BER expression for the FSO system adopting IM/DD and OOK is generally given by[3]

$$P_b(I) = Q\left(\frac{\eta I}{\sqrt{2N_0}}\right), \tag{3}$$

where $Q(\cdot)$ is the Gaussian Q-function[7]. Then, the existing exact closed-form expression of the average BER is obtained as[3]

$$\overline{P}_b^{\text{Ex.}} = \int_0^\infty Q\left(\frac{\eta I}{\sqrt{2N_0}}\right)e^{-I}dI = \frac{1}{2\pi}G_{3,2}^{2,2}\left[\bar{\gamma}\,\middle|\,\begin{matrix}1/2,0,1\\0,1/2\end{matrix}\right], \tag{4}$$

where the Meijer's G-function[8] $G_{p,q}^{m,n}\left[z\,\middle|\,\begin{matrix}a_1,\ldots,a_n,a_{n+1},\ldots,a_p\\b_1,\ldots,b_m,b_{m+1},\ldots,b_q\end{matrix}\right]$ is a generalization of the generalized hypergeometric function, which can be reduced to simpler special functions in many common cases, and can be defined by the following contour integral representation

$$G_{p,q}^{m,n}\left[z\,\middle|\,\begin{matrix}a_1,\ldots,a_n,a_{n+1},\ldots,a_p\\b_1,\ldots,b_m,b_{m+1},\ldots,b_q\end{matrix}\right] = \frac{1}{j2\pi} \times \int \frac{\Pi_{i=1}^{m}\Gamma(b_i - s)\,\Pi_{i=1}^{n}\Gamma(1-a_i+s)z^s}{\Pi_{i=m+1}^{q}\Gamma(1-b_i+s)\,\Pi_{i=n+1}^{p}\Gamma(a_i - s)}ds, \tag{5}$$

with the gamma function[8] $\Gamma(z) = \int_0^\infty t^{z-1}\exp(-\mathrm{t})\mathrm{dt}$. It is noteworthy that although the existing BER formula in Eq. (4) is given in closed-form in terms of the Meijer's G-function, it does not allow us to get intuitive and useful insights into the achievable BER performance as a function of SNR $\bar{\gamma}$, such as achievable asymptotic modulation gain and diversity order at high SNR, which will be examined in the following section.

Ⅳ. Novel Exact and Asymptotic BER Formulas

In an effort to overcome the shortcomings of the existing BER formula, we concentrate on the following relation about the Meijer's G-function given in Eq. (07.34.03.0897.01)[8]

$$\begin{aligned} G_{3,2}^{2,2}\left[z \left| \begin{matrix} a_1, a_2, a_3 \\ b_1, b_2 \end{matrix} \right.\right] &= \pi \csc(\pi(a_1 - a_2)) \left[\Gamma(1-a_1+b_1)\Gamma(1-a_1+b_2)\right. \\ &\quad \times z^{a_1-1} {}_2\widetilde{F}_2(1-a_1+b_1, 1-a_1+b_2; 1-a_1+a_2, \\ &\quad 1-a_1+a_3; 1/z) - \Gamma(1-a_2+b_1)\Gamma(1-a_2+b_2) \\ &\quad \times z^{a_2-1} {}_2\widetilde{F}_2(1-a_2+b_1, 1-a_2+b_2; a_1-a_2+1, \\ &\quad \left. 1-a_2+a_3; 1/z)\right], \end{aligned} \tag{6}$$

where $a_2 - a_1 \notin Z$ and ${}_p\widetilde{F}_q(a_1, \ldots, a_p; b_1, \ldots, b_q; z)$ is the regularized generalized hypergeometric function given in Eq. (07.32.02.0001.01)[8]. Then, by further utilizing Eq. (07.32.26.0001.01)[8], we can finally obtain the following novel exact closed-form formula of the average BER, after some straightforward algebraic manipulations, as

$$\begin{aligned} \overline{P}_b^{\text{Proposed}} &= \frac{1}{\sqrt{\pi \overline{\gamma}}} {}_2F_2\left(\frac{1}{2}, 1; \frac{1}{2}, \frac{3}{2}; \frac{1}{\overline{\gamma}}\right) \\ &\quad - \frac{1}{\overline{\gamma}} {}_2F_2\left(1, \frac{3}{2}; \frac{3}{2}, 2; \frac{1}{\overline{\gamma}}\right), \end{aligned} \tag{7}$$

where ${}_2F_2(a_1, a_2; b_1, b_2; z) = \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k z^k}{(b_1)_k (b_2)_k k!}$ is the generalized hypergeometric function given in Eq. (07.25.02.0001.01)[8] along with the Pochhammer symbol $(a)_k = \Gamma(a+k)/\Gamma(a)$. Note that the generalized hypergeometric function in Eq. (7) is a built-in function in most of the well-known mathematical software packages, such as Mathematica, MATLAB, etc.

Additionally, for $\overline{\gamma} \to \infty$ (i.e., $\frac{1}{\overline{\gamma}} \to 0$) and from ${}_2F_2(a_1, a_2; b_1, b_2; 0) = 1$ in Eq. (07.25.03.0001.01)[8], $\overline{P}_b^{\text{Proposed}}$ at sufficiently high SNR can be directly simplified to the asymptotic BER formula as

$$\overline{P}_b^{\text{Proposed}, \infty} = \frac{1}{\sqrt{\pi}} \overline{\gamma}^{-\frac{1}{2}}. \tag{8}$$

By using the fact that the asymptotic error rate can be expressed as $\overline{P}^{\infty} = (G^{\infty} \cdot \overline{\gamma})^{-d^{\infty}}$ in the high SNR regions[7], where G^{∞} and d^{∞} represent the asymptotic modulation gain and diversity order, respectively, it can be easily found that the achievable asymptotic modulation gain is $G^{\infty} = \pi$ and the corresponding asymptotic diversity order becomes $1/2$ (i.e., $d^{\infty} = 1/2$) for the FSO system under the strong atmospheric turbulence. Furthermore, it is also clear that whereas the existing BER formula given in Eq. (4) is expressed in terms of Meijer's G-function only and thus requires a complicated numerical contour integration, our derived formulas Eqs. (7) and (8) are more directly expressed as a function of the average SNR $\overline{\gamma}$ and thus they are much more effective at providing us useful insights into the achievable error rate performance of the FSO system (e.g., asymptotic modulation gain and diversity order).

V. Numerical Results

This section is devoted to the numerical evaluation of the error rate behavior of the FSO communication system employing IM/DD and OOK over strong atmospheric turbulence channels. To be specific, we attempt to validate the accuracy of the derived BER formulas (i.e., Eqs. (7) and (8)) through some numerical results.

Fig. 1 shows the average BER performances (i.e., $\overline{P}_b$) versus the average SNR $\overline{\gamma}$. As apparently shown in the figure, the numerical results for the average BER obtained from the proposed novel formula (i.e, Eq. (7)) exactly match with those obtained from the existing formula (i.e., Eq. (4)) in all SNR regions, which demonstrates that our derived formula is exactly the same as the existing formula, and this is natural because our formula is obtained by efficiently simplifying the existing one. In addition, the line from the derived asymptotic BER formula (i.e., Eq. (8)) tightly converges to the exact BER curve, especially in the middle and high SNR regions (cf. see the zoomed figure at the left bottom corner of

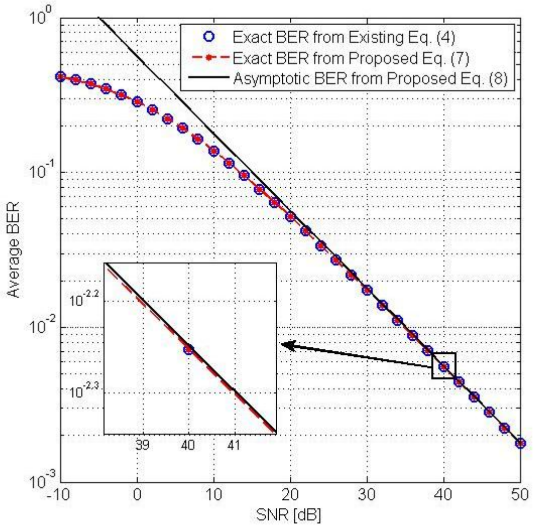

그림 1. 강한 대기 난기류 채널 환경에서 IM/DD와 OOK 기반의 FSO 시스템의 평균 비트 오류율 성능
Fig. 1. Average BER vs. SNR of FSO system with IM/DD and OOK over strong atmospheric turbulence channels.

Fig. 1). Furthermore, the figure clearly shows that the asymptotic diversity order is 1/2 (i.e., $d^{\infty}=1/2$) from the magnitude of the slope of the average BER versus SNR on a log-log scale at high SNR. Thus, we can finally verify the accuracy of the derived formulas, which implies that the proposed analytical results help us to accurately evaluate and predict the achievable BER performance of the FSO system, including asymptotic modulation gain and diversity order, over strong atmospheric turbulence environments.

VI. Conclusions

We have derived the novel exact and asymptotic closed-form formulas of the average BER for the FSO system with IM/DD and OOK under the strong atmospheric turbulence. We have also demonstrated that compared to the existing BER expression, the proposed formulas are more intuitive and provide more effective insights into the error rate behavior of the system. Moreover, we have further explicitly corroborated the exactness and efficiency of the derived BER expressions through numerical results.

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