Journal of The Institute of Electronics and Information Engineers Vol.62, NO.1, January 2025 https://doi.org/10.5573/ieie.2025.62.1.63
ISSN 2287-5026(Print) / ISSN 2288-159X(Online)

논문 2024-62-1-8

Nakagami-m 페이딩 채널에서 Dual-carrier Modulation (DCM) 기법의 점근적 비트오류율 분석

(Asymptotic BER Analysis of Dual-carrier Modulation (DCM) Scheme over Nakagami-m Fading Channel)

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요 약

본 논문에서는 Nakagami-m 페이딩 채널에서 Dual-carrier modulation (DCM) 기법의 비트오류율을 높은 신호 대비 잡음 비 수준에서 새로운 형태의 점근적 분석을 수행하였다. 나아가, 해당 상황에서 달성가능한 변조 이득 및 다이버시티 오더를 구하고, 멀리 떨어져 짝을 이루는 두 부반송파의 채널 환경이 다를 때 생기는 상대적 신호 대비 잡음 비 이득에 대해서도 제시하고 분석하였다. 다양한 페이딩 환경에 대한 수치적인 분석을 통해 본 논문에서 유도한 수식들이 실제 점근적 성능에 정확하게 근사함을 확인하였고, 이를 통해 실제 무선 통신 환경에서 DCM 기법을 최적화하여 구현하는데 도움을 줄 수 있을 것으로 보인다.

Abstract

This paper derives a novel asymptotic bit error rate (BER) expression for dual-carrier modulation (DCM) over Nakagami-m fading channels at high signal-to-noise ratio (SNR). Through this analysis, the achievable modulation gain and diversity order are provided, and the relative SNR gain that occurs when the channel environments of two distantly paired sub-carriers differ is also evaluated. Numerical results demonstrate that the derived expressions accurately predict the asymptotic behavior at high SNR across various fading scenarios. These findings offer valuable insights for optimizing DCM implementations in practical wireless communication environments.

Keywords : Dual-carrier modulation (DCM), Nakagami-m fading, Bit error rate (BER), Asymptotic analysis

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※ This research was financially supported by Hansung University.
Received ; October 21, 2024 Revised ; October 30, 2024
Accepted ; November 14, 2024

I. Introduction

Dual-carrier modulation (DCM) scheme was initially introduced in Ultra-Wideband (UWB) communication systems to improve spectral utilization through utilizing fading channel diversity[1, 2]. The main feature of DCM lies in combining two QPSK symbols into two 16-QAM symbols, which are then transmitted on widely separated sub-carriers to achieve diversity gain. Previous papers have analyzed the bit error rate (BER) of DCM in environments such as Rayleigh fading or Nakagami-m fading channels[3]. The Nakagami-m fading model is particularly significant because, by adjusting the parameter m, it can represent a wide range of fading conditions—from severe fading (small m) to no fading (large m)—making it well-suited to modeling real-world wireless environments[4].

While earlier works focused on BER assesment, our paper derives the efficient closed-form formula of the asymptotic BER for DCM at high SNR through asymptotic error rate analysis in Nakagami-m fading channels. Analyzing system performance at high SNR is crucial as it reveals the fundamental limits and provides insights into the asymptotically achievable performance of the communication scheme[5]. Specifically, based on the derived expression, we are able to obtain the asymptotically achievable modulation gain and diversity order as well as the relative SNR gain. Some numerical results are provided to corroborate the efficiency and accuracy of the derived formulas for various combinations of system and channel conditions.

Ⅱ. System Models

1. Dual-carrier modulation

For dual-carrier modulation, we first need to convert 200 bits into 100 QPSK symbols. The divided QPSK signals are then paired into two groups, resulting in 50 QPSK symbol pairs. At this point, the paired symbols $s_{i,} s_j$ are mapped to two 16-QAM symbols $x_i \in C_{1,}\ x_j \in C_2$[2] as described in Fig. 1, following the method described by the equation below,

$$\begin{bmatrix} x_i \\ x_j \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} s_i \\ s_j \end{bmatrix}. \qquad (1)$$

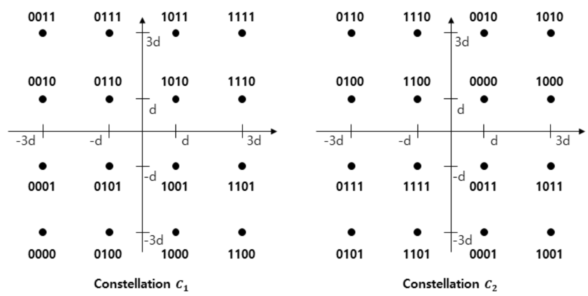

Fig. 1. Bit-to-symbol mappings for DCM.

To decode the received signal $y_{i,} y_j$ the DCM encoded as described above, the maximum likelihood (ML) criterion is applied as below

$$\min_{x_i \in C_1, x_j \in C_2} \left[|y_i - h_i x_i|^2 + |y_i - h_j x_j|^2 \right], \qquad (2)$$

where h_i is Nakagami-m fading channel of i-th sub-carrier and additive white Gaussian noise (AWGN) is considered.

Ⅲ. Performance Analysis

1. Existing BER performance analysis

The BER analysis was performed through the accumulation of the C_1 and C_2 constellations[3], and considering all possible cases, the combinations are divided into four categories: x_A, x_B, x_C, and x_D. These correspond to $d + j3d$, $-3d + jd$, $-d - j3d$,and $3d - jd$, respectively. Then, the tight upper bound of BER can be formulated as[3]

$$\begin{aligned} \overline{P}_b(E) &\approx 1/2\,\overline{P}_s(E) \\ &\le 1/2(\overline{P}_{x_A \to x_B} + \overline{P}_{x_A \to x_C} + \overline{P}_{x_A \to x_D}). \end{aligned} \qquad (3)$$

where $\overline{P}_{x_A \to x_\delta}$ is the average error probability of decoding $\delta \in \{B, C, D\}$ when A was transmitted. Then, for m_1, m_2, which are the Nakagami-m parameters, the average error probability can be derived as

$$\overline{P}_{x_A \to x_\delta} = \frac{1}{\pi} \int_0^{\pi/2} \prod_{\ell=1}^{2} \left(\frac{m_\ell}{m_\ell + a_{L,\ell}/\sin^2\phi} \right)^{m_\ell} d\phi, \quad (4)$$

where $a_{L,\ell} = \mu_{L,\ell} \overline{\gamma_\ell}/2$ and $(\mu_{L,1},\ \mu_{L,2})$ are given by $(8/5, 2/5)$, $(2/5, 18/5)$, and $(2/5, 8/5)$ depending on the symbol distance[3].

Furthermore, we can reformulate (2) with a similar method of previous literature[6] as below,

$$\begin{aligned} &\overline{P}_{x_A \to x_\delta} \\ &= \frac{1}{\pi} \int_0^{\pi/2} \left(\frac{m_1}{m_1 + \frac{a_{L,1}}{\sin^2\phi}} \right)^{m_1} \left(\frac{m_2}{m_2 + \frac{a_{L,2}}{\sin^2\phi}} \right)^{m_2} d\phi \\ &= \frac{B\left(m_1 + m_2 + \frac{1}{2}, \frac{1}{2}\right)}{2\pi \left(1 + \frac{a_{L,1}}{m_1}\right)^{m_1} \left(1 + \frac{a_{L,2}}{m_2}\right)^{m_2}} \\ &\times F_1\left(\frac{1}{2}, m_1, m_2, m_1 + m_2 + 1; \frac{1}{1 + \frac{a_{L,1}}{m_1}}, \frac{2}{1 + \frac{a_{L,2}}{m_2}} \right) \end{aligned} \qquad (5)$$

where $B(\cdot,\cdot)$ and $F_1(\cdot,\cdot,\cdot,\cdot;\cdot,\cdot)$ are the beta function[7] and the Appell hypergeometric function[7], respectively. The equation derived in this way aligns well with the BER equation from the previous paper[8], considering the definition of beta function[7].

2. Asymptotic BER performance analysis

At high SNR, i.e., $a_{L,\ell} \to \infty$, the two terms at the end of (5) approach 0, and as a result, the value of hypergeometric function becomes 1, which means

$$\begin{aligned} \overline{P}^{\infty}_{x_A \to x_\delta} &= \frac{B\left(m_1+m_2+\frac{1}{2},\frac{1}{2}\right)}{2\pi\left(\frac{a_{L,1}}{m_1}\right)^{m_1}\left(\frac{a_{L,2}}{m_2}\right)^{m_2}} \\ &= \frac{B\left(m_1+m_2+\frac{1}{2},\frac{1}{2}\right)}{2\pi\left(\frac{\mu_{L,1}}{2m_1}\right)^{m_1}\left(\frac{\mu_{L,2}}{2m_2}\right)^{m_2}} \\ &= \frac{m_1^{m_1} m_2^{m_2} 2^{m_1+m_2} B\left(m_1+m_2+\frac{1}{2},\frac{1}{2}\right)}{2\pi \mu_{L,1}^{m_1}\mu_{L,2}^{m_2}\overline{\gamma_1}^{m_1}\overline{\gamma_2}^{m_2}}. \end{aligned} \tag{6}$$

Without loss of generality, we can set the relative SNR ration between $\overline{\gamma_1}$ and $\overline{\gamma_2}$ as $k\overline{\gamma_1} = \overline{\gamma_2} = \overline{\gamma_{\rm av}}/2\eta$ for some $k,\eta \in \mathbb{R}$. Then, the asymptotic upper bound $\overline{P}_b^{U,\infty}(E)$ can be derived as

$$\begin{aligned} &\overline{P}_b^{U,\infty}(E) \\ &= \underbrace{\frac{m_1^{m_1} m_2^{m_2} B\left(m_1+m_2+\frac{1}{2},\frac{1}{2}\right)}{4\pi k^{m_2}\eta^{m_1+m_2}} \sum_{L=1}^{3} \mu_{L,2}^{-m_1}\mu_{L,2}^{-m_2}}_{I_{\rm DCM}} \\ &\quad \times \frac{1}{\overline{\gamma_{\rm av}}^{m_1+m_2}}. \end{aligned} \tag{7}$$

Since the asymptotic error rate at high SNR can be expressed as $\overline{P}^{\infty}(E) \approx \left[G \cdot \overline{\gamma}\right]^{-d}$ where G and d are the asymptotic modulation gain and diversity order[6], (7) yields the corresponding achievable modulation gain of $G_{\rm DCM} = I_{\rm DCM}^{-1/(m_1+m_2)}$ and the asymptotic diversity order of $d_{\rm DCM} = m_1 + m_2$.

In addition, it is also possible to formulate the relative SNR gain for the case of $k > 1$ compared to the case where the average SNRs of the two channels are equal (i.e., $k=1$), after some straightforward manipulations, as

$$\begin{aligned} {\rm SNR_{gain}}[{\rm dB}] &= 10\log_{10}\left[\frac{G_{\rm DCM}(k>1)}{G_{\rm DCM}(k=1)}\right] \\ &= \frac{10m_2}{m_1+m_2}\log_{10}k. \end{aligned} \tag{8}$$

Ⅳ. Numerical Results

In order to verify the accuracy of the asymptotic BER formula derived in the previous section, we evaluate the BER performance of DCM over Nakagami-m fading channels. Fig. 2 shows the numerical results with $m_1 = m_2$ and $\overline{\gamma_1} = \overline{\gamma_2}$.

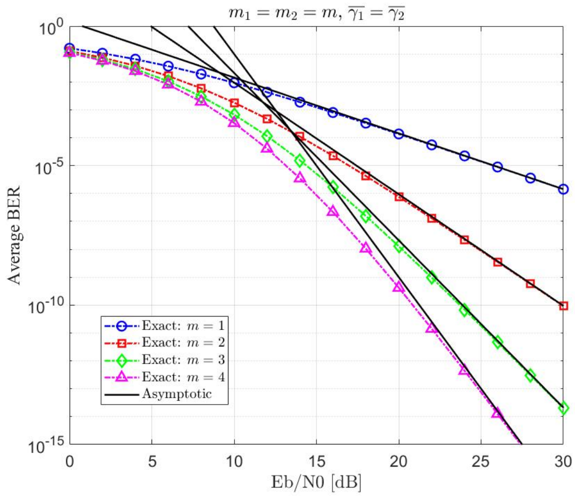

Fig. 2. Average and asymptotic BER performance of DCM for $m_1 = m_2 \in \{1,2,3,4\}$ and $\overline{\gamma_1} = \overline{\gamma_2} = \overline{\gamma_{\rm av}}$.

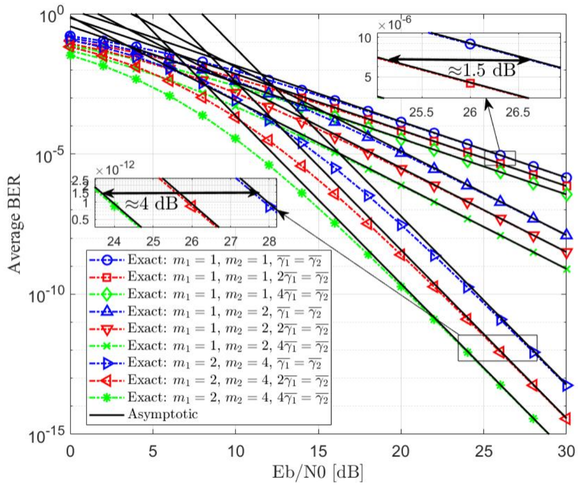

Fig. 3. Average and asymptotic BER performance of DCM for various values of m_1, m_2, and k.

As can be seen from the Fig. 2, the asymptotic results from the derived BER formula in (7) apparently show the tightness at high SNR for various fading scenarios.

In Fig. 3, the exact average BER curves and asymptotic BER lines, which are obtained from (5) and (7), respectively, are demonstrated for various combinations of m_1, m_2, and k. For example, in the case of $m_1 = m_2 = 1$, and $k = 2$, the relative SNR gain of 1.5051 [dB] from (8) is clearly observed from the upper zoomed figure. The lower zoomed figure also corroborates the accuracy of (8) for the case of $m_1 = 2, m_2 = 4$, and $k = 4$, showing the relative SNR gain of 4.0137 [dB].

V. Conclusions

In this paper, we have derived a closed-form asymptotic BER expression for DCM scheme over Nakagami-*m* fading channels. Through some numerical results, we have demonstrated that the derived expression accurately fits the asymptotic behavior. We have also evaluated the achievable modulation gain, diversity order, and relative SNR gain for various system and channel parameters, providing valuable insights for optimizing DCM implementations in practical wireless communication environments. By revealing the fundamental performance limits and quantifying potential gains at high SNR, our analysis can provide essential guidance for designing more resilient and efficient communication systems that can better cope with varying fading conditions.

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